

T H E
D I S T A N C E
O F T H E
S U N from the E A R T H
D E T E R M I N E D,
B Y T H E
T H E O R Y of G R A V I T Y.

Together with several other Things relative
to the same Subject.

By Dr M A T T H E W S T E W A R T, *K*

Professor of Mathematics in the Univerfity of Edinburgh.

Being a fupplement to *Traëts physical and mathe-
matical*, lately published by the fame author.

E D I N B U R G H:

Printed for A. MILLAR, J. NOURSE, and D. WILSON,
London; and W. SANDS, and A. KINCAID & J. BELL,
Edinburgh.

MDCCLXIII.



T O

THE RIGHT HONOURABLE,

JOHN EARL OF BUTE,

ONE OF HIS MAJESTY'S MOST
HONOURABLE PRIVY COUNCIL,

KNIGHT OF THE MOST NOBLE
ORDER OF THE GARTER,

&c. &c. &c.

A DISTINGUISHED PATRON OF ALL
THE ELEGANT AND USEFUL ARTS
AND SCIENCES, AND AN EMINENT
PROFICIENT IN THEM,

THE FOLLOWING SHEETS ARE HUM-
BLY INSCRIBED BY

HIS LORDSHIP'S
MOST OBEDIENT AND
DEVOTED SERVANT,

MATTH. STEWART.

THE RIGHT HONOURABLE

JOHN EARL OF BUCKINGHAM

ONE OF HIS MAJESTY'S MOST
HONOURABLE PRIVY COUNCIL

KNIGHT OF THE MOST NOBLE
ORDER OF THE GARTER



A DISTINGUISHED PERSON OF
THE REGENT AND USEFUL ARTS
AND SCIENCES AND AN EMINENT
PROFICIENT IN THEM

THE FOLLOWING CERTIFICATE
IS HEREBY

HIS LORDSHIP
MOST GRACIOUSLY
AND BENEFICENTLY

HAS BEEN

P R E F A C E.

TO determine the distance of the sun from the earth, is a problem of great antiquity, probably as old as the science of astronomy itself. It has justly been esteemed a problem of the utmost difficulty, and a very curious one. It being solved, the magnitude of the primary planets, and the extent of the solar system, are easily ascertained. Various methods have been made use of, by astronomers in different ages, to solve it; but the immense distance of the sun from the earth rendered all these methods abortive. The celebrated Dr Halley proposed a very ingenious method of solving this problem, by observing the transit of Venus over the disk of the sun in the year 1761; but this method has not answered expectation.

The author of the following sheets was early of opinion, that this method would fail; and therefore, for many years past, he has employed

a great part of his time to find out, if possible, another method of considering the subject. This gave rise to the Tracts physical and mathematical lately published by him. In these tracts a method was pointed out for determining the sun's distance from the earth, upon the supposition, that the solar force affecting the gravity of the moon to the earth could be ascertained; which he thought might be done, either from the motion of the moon's apogee, or from the motion of its nodes. He expected that this would be done by an abler hand: but as he has seen nothing of this kind done, after near two years publication of these tracts, he thought proper to consider this subject himself; and has ascertained the solar force affecting the gravity of the moon to the earth, and from that has calculated very accurately the mean distance of the sun from the earth.

In his preface to the aforementioned tracts he affirmed,

affirmed, that as the various methods hitherto made use of, to determine the sun's distance, had failed on account of the immense distance of the sun from the earth, the method he proposed would be the more accurate the more distant the sun is from the earth. The intelligent reader will soon be satisfied, that this assertion is justly founded.

The author has now considered two of the most difficult problems in astronomy, namely, Kepler's problem *, and the ascertaining the sun's

* That this problem was esteemed by the great Kepler, the proposer of it, a very difficult problem, is evident from the following passage, in the 300th page of his book, intitled, *De motibus stellæ Martis*.

Mihi sufficit credere, solvi a priore non posse, propter arcus et sinus irrationales. Erranti mihi, quicumque viam monstraverit, is erit mihi magnus Apollonius.

This problem, it is well-known, is of great importance to astronomy, and has been solved by several mathematicians of great note : but they have had recourse to the more difficult parts of mathematics ; whereas the solution given of it in the aforementioned tracts, requires only the knowledge of plain trigonometry.

distance

distance from the earth. There remains another to be considered of great importance, and equally difficult; that is, to determine the lunar path. This is a very complex problem, and would require more time and leisure to consider it than can be expected from one in the author's situation. However, he has already made some advances in it, which he hopes in due time will be published. If any incline to consider this subject, they may find some propositions in the following sheets that will be of considerable use.

College of Edinburgh,
July 19. 1763.

THE
DISTANCE
OF THE
SUN from the EARTH
DETERMINED.

SECT. I.

The Solar Force affecting the Gravity of the Moon to the Earth.

PROP. I. *Fig. I.*

LET the circle ABC represent the orbit of the moon round the earth at T, ATC the syzigial line, and TB the line of the quadratures; in TA take TD
A *so*

so that the square of TA may be triple the square of TD; with the centre T, and distance TD, describe the circle DEF; upon AT describe the semicircle AHT; in TA produced, take TG so that TG may be to TD as the periodic time of the earth round the sun to the periodic time of the moon round the earth; and with the centre T, and distance TG, describe the circle GRK; suppose the moon at any point L in its orbit; and let La be an indefinitely little part of the orbit; join TL, Ta, meeting the circle GRK in M, d, the semicircle AHT in N, b, and the circle DEF in o, c: the centripetal force of the moon to the earth will be to the solar force affecting the gravity of the moon to the earth at the point L, nearly as the sector TMd to the space Nacb.

Draw

Prop. I. THE SOLAR FORCE. 3

Draw LX perpendicular to AC, meeting AC in X; join AN; and let TL meet the circle DEF in P; let TV be to TA in the duplicate ratio of the periodic time of the earth round the sun to the periodic time of the moon round the earth; and let the circle DEF meet TC in F.

The triangles ATN, LTX are every way equal, because the angles ANT, LXT are both right, and the angle ATL common; and likewise TA, TL are equal; therefore TN, TX are equal; therefore the squares of TN, TX are equal: but the squares of T ϕ , TD are likewise equal, because T ϕ , TD are equal; therefore the rectangles oNP, DXF are equal: because the sector TM ϕ is to the sector TN ϕ as the square of TM to the square of TN, and the

sector TNb is to the space $Nocb$ as the square of TN to the rectangle oNP ; therefore the sector TMd is to the space $Nocb$ as the square of TM to the rectangle oNP ; that is, as the square of TG to the rectangle DXF . Again, because TV is to TA in the duplicate ratio of the periodic time of the earth round the sun to the periodic time of the moon round the earth; that is, in the duplicate ratio of TG to TD ; therefore TV will be to TA as the square of TG to the square of TD : but the rectangle VTA is to the square of TA as TV to TA ; therefore the rectangle VTA is to the square of TA as the square of TG to the square of TD : and because the square of TA is triple the square of TD ; therefore the rectangle VTA is triple the square of TG ; therefore the rectangle VTA will be to thrice the rectangle

angle

angle DXF as thrice the square of TG to thrice the rectangle DXF; that is, as the square of TG to the rectangle DXF; that is, as the sector TMD to the space Nacb: but the centripetal force of the moon to the earth is [Tract 4. prop. 7.] to the solar force affecting the gravity of the moon to the earth at the point L, as the rectangle VTA to thrice the rectangle DXF; therefore the centripetal force of the moon to the earth is to the solar force affecting the gravity of the moon to the earth at the point L, as the sector TMD to the space Nacb. Q. E. D.

C O R O L L A R Y.

Let the semicircle AHT meet the circle DHE in the point H; join TH meeting the circle AQB in Q; if the point L be in the
arc

arc AQ, the solar force diminishes the gravity of the moon to the earth; and if the point L be in the arc QB, the solar force increases the gravity of the moon to the earth.

Join AH, QD. Because AT, TH are equal to QT, TD, and the angle ATQ common; therefore the angles QDT, AHT are equal: but the angle AHT is a right angle, because an angle in a semicircle; therefore the angle QDT is likewise a right angle; therefore, if the point L be in the arc AQ, it is evident, that the point X falls without FD; and therefore the solar force diminishes the gravity of the moon to the earth. The same way it is shown, that if the point L be in the arc QB, the solar force increases the gravity of the moon to the earth.

P R O P.

PROP. II. Fig. 1.

The same construction remaining as in the former proposition. Let the circles GRK, DHE meet TB in K, E: if the quadrant GTK represent the sum of the centripetal forces that have acted upon the moon while it described the quadrant AQB, the portion of the semicircle AHT that falls without the quadrant TDE, will represent the sum of the solar forces diminishing the gravity of the moon to the earth while it described the arc AQ; and the portion of the quadrant TDE that falls without the semicircle AHT will represent the sum of the solar forces increasing the gravity of the moon to the earth while it described the arc QB.

Let

Let the arc AQ be divided into indefinitely little parts, Af , fg , &c.; join Tf , Tg , &c.; let Tf meet the circle DHE in h , and the semicircle AHT in k , and Tg meet the circle DHE in l , and the semicircle AHT in m , and so on; let Tf , Tg , &c. meet the circle GRK in n , p , &c.; let Y be any point in the arc TH .

Because the arcs Af , fg , &c. are indefinitely little, the centripetal force of the moon to the earth will [1.] be to the solar force diminishing the gravity of the moon to the earth at the point A , as the sector TGn , to the space $ADbk$; therefore, if the sector TGn represent the centripetal force of the moon to the earth, the space $ADbk$ will represent the solar force diminishing the gravity of the moon to the earth at the point

Prop. 2. THE SOLAR FORCE. 9

point A. The same way it is shown, that if the sector Tnp represent the centripetal force of the moon to the earth, the space $kblm$ will represent the solar force diminishing the gravity of the moon to the earth at the point f , and so on; but the sectors TGn , Tnp , &c. make up the sector TGR , and the spaces $ADbk$, $kbfm$, &c. make up the space AHD ; therefore if the sector TGR represent the sum of the centripetal forces of the moon to the earth that have acted upon the moon while the moon described the arc AQ , the space AHD will represent the sum of the solar forces diminishing the gravity of the moon to the earth while the moon described the arc AQ . The same way it is shown, that if the sector TRK represent the sum of the centripetal forces of the moon to the earth that have

B acted

acted upon the moon while the moon described the arc QB, the space TYHE will represent the sum of the solar forces increasing the gravity of the moon to the earth while the moon described the arc QB; and therefore it is evident, that if the quadrant TKG represent the sum of the centripetal forces of the moon to the earth that have acted upon the moon while the moon described the quadrant AQB, the portion of the semicircle AHT that falls without the quadrant TDE will represent the sum of the solar forces diminishing the gravity of the moon to the earth while the moon described the arc AQ; and the portion of the quadrant TDE that falls without the semicircle AHT will represent the sum of the solar forces increasing the gravity of the moon

Prop. 2. THE SOLAR FORCE. II

moon to the earth while the moon described the arc QB. *Q. E. D.*

C O R O L L A R Y.

Let there be any semicircle AHT; in TA take TD, so that the square of TA may be triple the square of TD; draw TE perpendicular to TA; with the centre T, and distance TD, describe the circle DHE, meeting TE in E, and the semicircle in H: the sum of the solar forces diminishing the gravity of the moon to the earth will be to the sum of the solar forces increasing the gravity of the moon to the earth, as the portion of the semicircle AHT that falls without the quadrant TDE, to the portion of the quadrant TDE that falls without the semicircle AHT.

B 2

PROP.

PROP. III. Fig. 2.

Let there be a semicircle whose diameter is AB; in AB take BC so that the square of BA may be triple the square of BC; draw BD perpendicular to BC; with the centre B, and distance BC, describe a circle meeting BD in D, and the semicircle in E; and join DC: the portion of the semicircle that falls without the quadrant will be to the portion of the quadrant that falls without the semicircle, as DC, together with the arc CE, to the excess of DC above the arc DE.

Let F be the centre of the semicircle AEB; join BE, FE, AE; draw FG parallel to BE, meeting the semicircle in G, and AE in H; and draw FK parallel to AE, meeting the semicircle in K, and BE in L.

The

Prop. 3. THE SOLAR FORCE. 13

The angle AEB is a right angle, because an angle in a semicircle; therefore the angle AHF is a right angle; therefore the sector AFE is double of the sector AFG. The same way it is shown, that the sector BFE is double of the sector BFK. Because the angle AFG is equal to the angle CBE, the sectors AFG, CBE will be similar; therefore the sector AFG will be to the sector CBE as the square of AF to the square of BC; therefore the sector AFE will be to the sector BCE as twice the square of AF to the square of BC; that is, because the square of AB is triple the square of BC, as three to two; therefore, by division, the excess of the sector AFE above the sector CBE will be to the sector CBE as one to two: but the excess of the sector AFE above the sector CBE is equal
to

to the excess of the space AGE $\bar{C}$ above the triangle FEB; therefore the excess of the space AGE $\bar{C}$ above the triangle FEB is to the sector CBE as one to two; and therefore the excess of the space AGE $\bar{C}$ above the triangle FEB is equal to half the sector CBE; therefore the space AGE $\bar{C}$ is equal to half the sector CBE, together with the triangle FEB. Because the square of AB is triple the square of BC, or BE; therefore the sum of the squares of AE, EB is triple the square of BE; therefore the square of AE is double the square of BE, or BC: but the square of DC is double the square of BC; therefore AE, DC are equal. Because the rectangle AEB is double of the triangle AEB; therefore the rectangle DCB is double of the triangle AEB; and therefore the rectangle DCB is quadruple of the triangle

triangle FEB; therefore the triangle FEB is equal to one fourth part of the rectangle DCB. Because the sector CBE is equal to one half of the rectangle contained by the arc CE and CB; therefore half the sector CBE will be equal to one fourth part of the rectangle contained by CB and the arc CE; therefore the space AGE C is equal to one fourth part of the rectangle contained by CB and the arc CE; together with one fourth part of the rectangle DCB; therefore four times the space AGE C is equal to the rectangle contained by CB and the arc CE, together with the rectangle DCB.

Again : Because FK is parallel to AE, the angle BFK will be equal to the angle BAE, that is, equal to the angle DBE ; therefore
the

the sectors BFK, EBD are similar; therefore the sector BFK will be to the sector EBD as the square of BF to the square of BD, or BC; therefore the sector EFB will be to the sector EBD as twice the square of BF to the square of BC, that is, as three to two; therefore, by division, the excess of the sector BFE above the sector EBD will be to the sector EBD as one to two: but the excess of the sector BFE above the sector EBD is equal to the excess of the triangle BFE above the space BKED; therefore the excess of the triangle BFE above the space BKED is to the sector BED as one to two; therefore the excess of the triangle BFE above the space BKED is equal to one half of the sector BED; therefore the space BKED is equal to the excess of the triangle BFE above one half of the sector BED:
but

but the triangle BFE is equal to one fourth part of the rectangle DCB, and half the sector BED is equal to one fourth part of the rectangle contained by BD, or BC, and the arc DE; therefore the space BKED will be equal to the excess of one fourth part of the rectangle DCB above one fourth part of the rectangle contained by BC and the arc DE; therefore four times the space BKED will be equal to the excess of the rectangle DCB above the rectangle contained by CB and the arc DE. Because four times the space AGECE is equal to the rectangle DCB, together with the rectangle contained by CB, and the arc CE, and four times the space BKED is equal to the excess of the rectangle DCB above the rectangle contained by CB and the arc DE; therefore four times the space AGECE will be to four times

C

the

the space BKED, as the rectangle DCB, together with the rectangle contained by CB, and the arc CE, to the excess of the rectangle DCB above the rectangle contained by CB, and the arc DE : but the rectangle DCB, together with the rectangle contained by CB, and the arc CE, is to the excess of the rectangle DCB above the rectangle contained by CB, and the arc DE, as DC, together with the arc CE, to the excess of DC above the arc DE ; therefore four times the space AGE^C will be to four times the space BKED, as DC, together with the arc CE, to the excess of DC above the arc DE ; and therefore the space AGE^C will be to the space BKED, as DC, together with the arc CE, to the excess of DC above the arc DE.

Q. E. D.

✱

P R O P.

P R O P. IV. Fig. 3.

Let TA be the syzigial line, and TB the line of the quadratures; and supposing the orbit of the moon round the earth at T to be a circle, let ACB be a quadrant of the lunar orbit; in TA take TD so that the square of TA may be triple the square of TD; draw DC perpendicular to TA, meeting the circle ACB in C; and join AB: the sum of the solar forces diminishing the gravity of the moon to the earth will be to the sum of the solar forces increasing the gravity of the moon to the earth, as BA, together with the arc AC, to the excess of AB above the arc BC.

Join TC; and let CE, a tangent to the

C 2

circle

circle at C, meet TA in E; and upon TE describe the semicircle TCE.

Because the angle TCE is a right angle, and likewise the angle CDT a right angle, ET will be to TC as TC to TD, that is, ET will be to TA as TA to TD; therefore the square of ET will be to the square of TA as the square of TA to the square of TD: but the square of TA is triple the square of TD; therefore the square of TE is triple the square of TA; and therefore [Cor. to Prop. 2.] the sum of the solar forces diminishing the gravity of the moon to the earth will be to the sum of the solar forces increasing the gravity of the moon to the earth, as the portion of the semicircle that falls without the quadrant to the portion of the quadrant that falls without the semicircle:

circle: but the portion of the semicircle that falls without the quadrant is [Prop. 4.] to the portion of the quadrant that falls without the semicircle, as BA, together with the arc AC, to the excess of BA above the arc BC; therefore the sum of the solar forces diminishing the gravity of the moon to the earth will be to the sum of the solar forces increasing the gravity of the moon to the earth, as BA, together with the arc AC, to the excess of BA above the arc BC, Q. E. D.

C O R O L L A R Y.

The number of degrees and minutes in an arc equal to the line AB, is $81^{\circ} 10'$, the number of degrees and minutes in the arc AB is $54^{\circ} 44'$, the number of degrees and minutes

minutes in the arc BC is $35^{\circ} 16'$; therefore the sum of the solar forces diminishing the gravity of the moon to the earth will be to the sum of the solar forces increasing the gravity of the moon to the earth as $135^{\circ} 54'$ to $45^{\circ} 54'$, that is, as 8154' to 2754', that is, as 151 to 51; therefore the sum of the solar forces diminishing the gravity of the moon to the earth is very nearly triple the sum of the solar forces increasing the gravity of the moon to the earth.

P R O P. V. Fig. 4. 5.

Let TA represent the mean distance of the moon from the earth; at T draw AB perpendicular to TA; and let TA be to AB in the duplicate ratio of the periodic time of the earth round the sun to the periodic time of

of the moon round the earth; in BA produced towards A take AC double of AB; in TA, or in TA produced towards A, take TD equal to the distance of the moon from the earth at any given time; and in BC take CE such that BC may be to CE in the duplicate ratio of the radius to the sine of the elongation of the moon from the syzigial line at the given time, that is, let BC be to CE in the duplicate ratio of the distance of the moon from the earth to its distance from the syzigial line; join TE, and draw DF parallel to BC meeting TE in F; let TA represent the centripetal force of the moon to the earth at the distance TA, and DF will represent the force of the sun affecting the gravity of the moon to the earth at the given time: if DF and AC be on the same side of TA, the force DF diminishes

diminishes the gravity of the moon to the earth; but if on opposite sides of TA, the force DF increases the gravity of the moon to the earth.

Let G be the true place of the moon at the given time, and TA the syzygial line; join TG; with the centre T, and distance TA, describe a circle meeting TG, TA in H, K; draw HL perpendicular to TA meeting TA in L; in TA take TM such that the square of TA may be triple the square of TM; in TK take TN equal to TM; and let the rectangle BAO be equal to the square of TA.

Because BC is to CE in the duplicate ratio of the radius to the sine of the elongation of the moon from the syzygial line,

BC

BC will be to CE as the square of TH to the square of HL; therefore, conversely, CB will be to BE as the square of TH to the square of TL: and because CB is triple of BA, and likewise the square of TH triple the square of TM, AB will be to BE as the square of TM to the square of TL; therefore, inversely, BE will be to AB as the square of TL to the square of TM; therefore, by division, EA will be to AB as the rectangle MLN to the square of TM; therefore, inversely, BA will be to AE as the square of TM to the rectangle MLN: but the rectangle BAO is to the rectangle OAE as BA to AE; therefore the rectangle BAO is to the rectangle OAE as the square of TM to the rectangle MLN: and because the rectangle BAO, that is, the square of TA, is triple the square

D

of

of TM ; therefore the rectangle OAE is triple the rectangle MLN : and because TA is to AE as the rectangle OAT to the rectangle OAE ; therefore TA will be to AE as the rectangle OAT to thrice the rectangle MLN . Because the rectangle BAO is equal to the square of TA , OA will be to AT as AT to AB : but AT is to AB in the duplicate ratio of the periodic time of the earth round the sun to the periodic time of the moon round the earth; therefore OA is to AT in the duplicate ratio of the periodic time of the earth round the sun to the periodic time of the moon round the earth; therefore, supposing the moon at the point H , the centripetal force of the moon to the earth at the point H will [Traët. 4. prop. 7.] be to the force of the sun affecting the gravity of the moon to the earth
at

at the point H, as the rectangle OAT to thrice the rectangle MLN; that is, as TA to AE; therefore if TA represent the centripetal force of the moon to the earth at the distance TA, or TH, AE will represent the force of the sun affecting the gravity of the moon to the earth at the point H: but the force of the sun affecting the gravity of the moon to the earth at the distance TH is [Tract. 4. prop. 18.] to the force of the sun affecting the gravity of the moon to the earth at the distance TD, or TG, as TH to TG; that is, as TA to TD; that is, as AE to DF; therefore, because AE represents the force of the sun affecting the gravity of the moon to the earth at the point H, DF will represent the force of the sun affecting the gravity of the moon to the earth at the point G. Q. E. D.

PROP. VI. Fig. 6.

Let ABCD represent the orbit of the moon round the earth at T; and suppose the orbit of the moon to be a circle, and to coincide with the plane of the ecliptic, let AC be the syzigial line; let the moon be at any point L in its orbit; draw LM perpendicular to AC, meeting AC in M; produce TM to H, and let TH be triple of TM; join LH, LT; let LH be to LK in the duplicate ratio of the periodic time of the earth round the sun to the periodic time of the moon round the earth: the solar force affecting the gravity of the moon to the earth acts in the direction LH; and the centripetal force of the moon to the earth will be to the solar force acting upon the moon in the direction LH as TL to LK.

Let

Prop. 6. THE SOLAR FORCE. 29

Let ST represent the distance of the sun from the earth; and let TV be to TA in the duplicate ratio of the periodic time of the earth round the sun to the periodic time of the moon round the earth.

It was shown in the sixth proposition of the fourth tract, that the solar force affecting the gravity of the moon to the earth acts in the direction LH ; and in the corollary to the same proposition it was shown, that the force of the earth to the sun is to the solar force acting upon the moon in the direction LH nearly as ST to LH : but the centripetal force of the moon to the earth is to the centripetal force of the earth to the sun [Prop. 5. Tract 4.] as TV to ST ; therefore, by equality, the centripetal force of the moon to the earth will be to the solar force

force acting upon the moon in the direction LH as TV to LH: but because TV is to TA as LH to LK, alternating, TV will be to LH as TA, or TL, to LK; therefore the centripetal force of the moon to the earth is to the solar force acting upon the moon in the direction LH as TL to LK.
Q. E. D.

P R O P. VII. Fig. 7.

The solar force affecting the gravity of the moon to the earth in any part of the lunar orbit may be resolved into two forces: the one diminishes the gravity of the moon to the earth; the other acts upon the moon in a line parallel to the line of the quadratures, and presses the moon towards the syzigial line. The centripetal force of the moon to
the

the earth is to half the solar force diminishing the gravity of the moon to the earth in the duplicate ratio of the periodic time of the earth round the sun to the periodic time of the moon round the earth. The solar force acting upon the moon in the direction parallel to the line of the quadratures is to the force diminishing the gravity of the moon to the earth, as thrice the distance of the moon from the syzигial line to twice the distance of the moon from the earth.

Let the circle ABCD represent the orbit of the moon round the earth at T; and let AC be the syzигial line, and BD the line of the quadratures; and let the moon be at any point L in its orbit; join TL; and draw LM perpendicular to TA, meeting TA in M; produce TM to H; and let
TH

TH be triple of TM; join LH; draw HE parallel to LM, meeting TL produced in E; and complete the parallelogram LEHF; let LH be to LK in the duplicate ratio of the periodic time of the earth round the sun to the periodic time of the moon round the earth; let LE be to LG as LH to LK; likewise let LF be to LN as LH to LK.

It was shown [Prop. 6.] that the solar force affecting the gravity of the moon to the earth acts in the direction LH; and that the centripetal force of the moon to the earth is to the solar force acting upon the moon in the direction LH, as TL to LK; let the force acting in the direction LH be resolved into two forces, one acting in the direction LE, the other acting in the direction LF. Because LH is to LK as LE

to

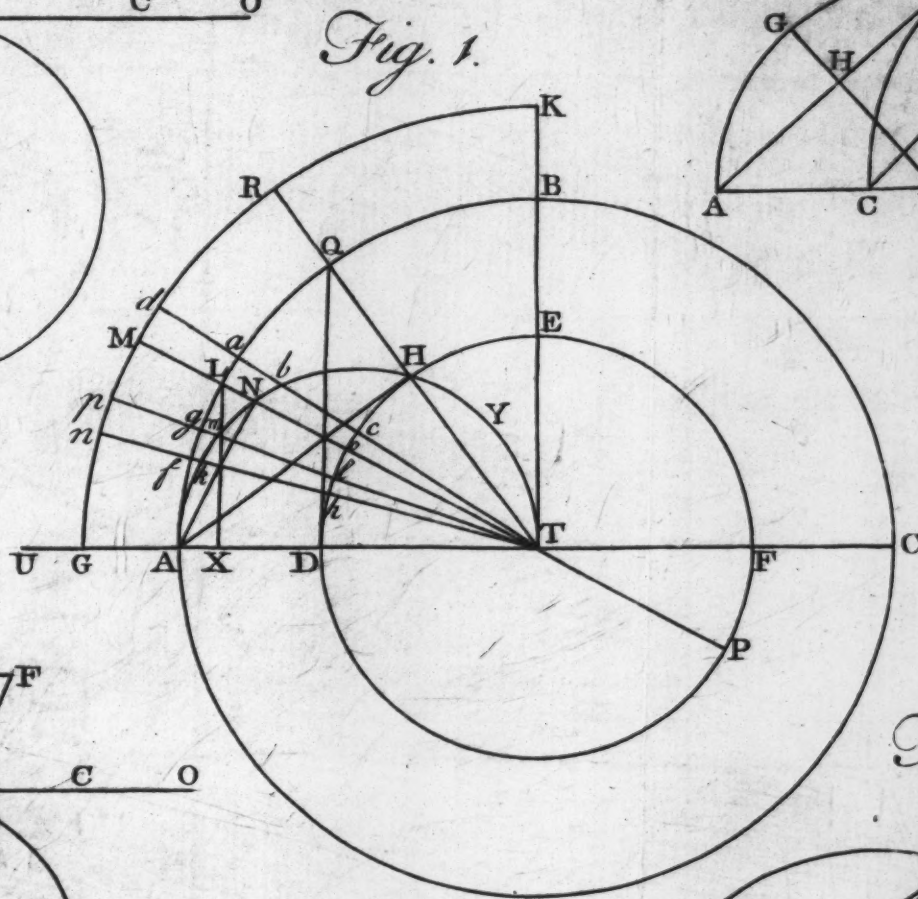
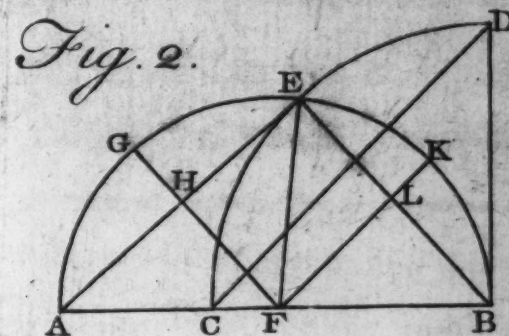
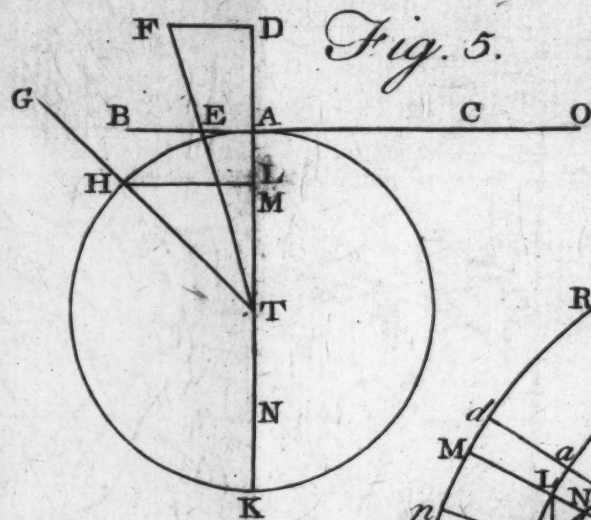
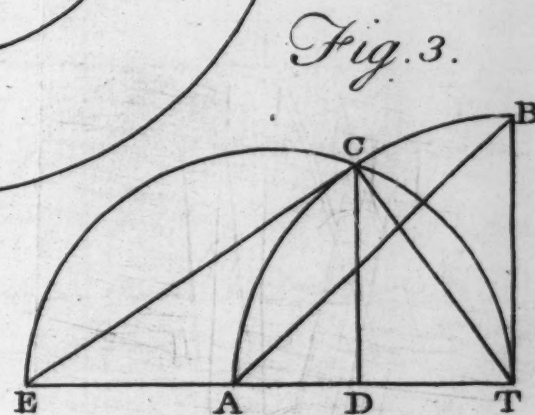
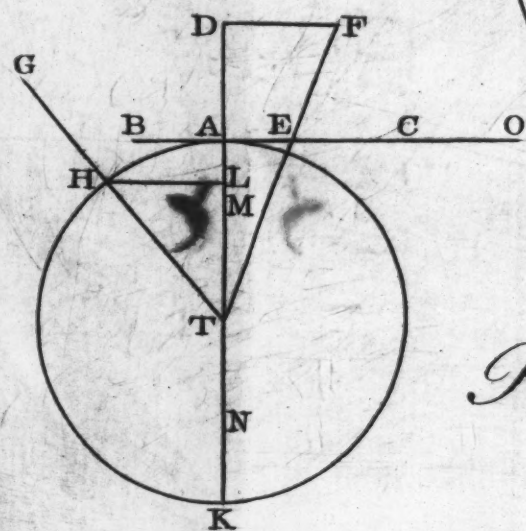


Fig. 4.



to LG, and likewise as LF to LN, if LK represent the solar force acting in the direction LH, LG, LN will represent the solar forces acting in the directions LE, LF; and the centripetal force of the moon to the earth will be to the solar force acting in the direction LE as TL to LG; likewise the centripetal force of the moon to the earth will be to the solar force acting in the direction LF as TL to LN; and the solar force acting in the direction LE will be to the solar force acting in the direction LF as LG to LN: but LE is to LG as LF to LN; therefore, alternating, LE is to LF as LG to LN; therefore the solar force acting in the direction LE is to the solar force acting in the direction LF as LE to LF. It is evident, that the solar force acting in the direction LE diminishes the gra-

E

vity

vity of the moon to the earth, and that the solar force acting in the direction LF presses the moon towards the syzigial line. Because TH is triple of TM, HE, or LF, will be triple of LM, and LE will be double of LT; therefore TL will be to the half of LG as LE to LG; that is, as LH to LK: but LH is to LK in the duplicate ratio of the periodic time of the earth round the sun to the periodic time of the moon round the earth; therefore TL is to the half of LG in the duplicate ratio of the periodic time of the earth round the sun to the periodic time of the moon round the earth: but because the centripetal force of the moon to the earth is to the solar force acting upon the moon in the direction LE as TL to LG, the centripetal force of the moon to the earth will be to half the solar force acting upon the moon

moon in the direction LE as TL to the half of LG; therefore the centripetal force of the moon to the earth will be to half the solar force diminishing the gravity of the moon to the earth in the duplicate ratio of the periodic time of the earth round the sun to the periodic time of the moon round the earth. Again, because the force acting in the direction LE is to the force acting in the direction LF as LE to LF, and LE is double of TL, and LF triple of LM; therefore the force diminishing the gravity of the moon to the earth will be to the force acting upon the moon in the direction parallel to the line of the quadratures, as twice the distance of the moon from the earth to thrice the distance of the moon from the syzigial line; and therefore the force acting upon the moon in the direction parallel

to the line of the quadratures is to the force diminishing the gravity of the moon to the earth, as thrice the distance of the moon from the syzigial line to twice the distance of the moon from the earth, *Q. E. D.*

C O R O L L A R Y.

It is evident from the proposition, that the forces diminishing the gravity of the moon to the earth, at different distances of the moon from the earth, are to one another as the distances of the moon from the earth; and likewise the forces acting upon the moon in the direction parallel to the line of the quadratures, at different distances of the moon from the syzigial line, are to one another as the distances of the moon from the syzigial line.

P R O P.

P R O P. VIII. Fig. 8.

The solar force affecting the gravity of the moon to the earth in any part of the lunar orbit, may be resolved into two forces; the one increases the gravity of the moon to the earth; the other acts upon the moon in a line parallel to the syzigial line, and pulls the moon from the line of the quadratures: the centripetal force of the moon to the earth is to the solar force increasing the gravity of the moon to the earth in the duplicate ratio of the periodic time of the earth round the sun to the periodic time of the moon round the earth: the solar force acting upon the moon in the direction parallel to the syzigial line is to the solar force increasing the gravity of the moon to the earth, as thrice the distance

distance of the moon from the line of the quadratures to the distance of the moon from the earth.

Let the circle ABCD represent the orbit of the moon round the earth at T; and let AC be the syzygial line, and BD the line of the quadratures; and let the moon be at any point L in its orbit; join TL; and draw LM perpendicular to TA, meeting TA in M; produce TM to H; and let TH be triple of TM; join LH; and complete the parallelogram THEL; let LH be to LK in the duplicate ratio of the periodic time of the earth round the sun to the periodic time of the moon round the earth; let TL be to LG as LH to LK; likewise let LE be to LN as LH to LK; and let LE meet BD in F.

It

It was shown [Prop. 6.] that the solar force affecting the gravity of the moon to the earth acts in the direction LH, and that the centripetal force of the moon to the earth is to the solar force acting upon the moon in the direction LH, as TL to LK. Let the force acting in the direction LH be resolved into two forces, one acting in the direction LT, the other acting in the direction LE. Because LH is to LK as LT to LG, and likewise as LE to LN, if LK represent the solar force acting in the direction LH, LG, LN will represent the solar forces acting in the directions LT, LE; and the centripetal force of the moon to the earth will be to the solar force acting in the direction LT as TL to LG; likewise the centripetal force of the moon to the earth will be to the force acting in the direction

LE

LE as TL to LN; and the solar force acting in the direction LT will be to the solar force acting in the direction LE as LG to LN: but TL is to LG as LE to LN; therefore, alternating, LT is to LE as LG to LN; therefore the solar force acting in the direction LT will be to the solar force acting in the direction LE as TL to LE. It is evident that the solar force acting in the direction LT increases the gravity of the moon to the earth, and that the solar force acting in the direction LE pulls the moon from the line of the quadratures. Because TH is triple of TM, LE will be triple of TM: but TM, LF are equal; therefore LE is triple of LF. Because the centripetal force of the moon to the earth is to the solar force increasing the gravity of the moon to the earth as TL to LG; that is, as LH

to

to LK; and LH is to LK in the duplicate ratio of the periodic time of the earth round the sun to the periodic time of the moon round the earth; therefore the centripetal force of the moon to the earth will be to the solar force increasing the gravity of the moon to the earth in the duplicate ratio of the periodic time of the earth round the sun to the periodic time of the moon round the earth. Again: Because the force acting in the direction LT is to the force acting in the direction LE as TL to LE, and LE is triple of LF; therefore the force increasing the gravity of the moon to the earth will be to the force acting upon the moon in the direction parallel to the syzигial line, as the distance of the moon from the earth to thrice the distance of the moon from the line of the quadratures; and therefore, in-

F

versely,

verfely, the force acting upon the moon in the direction parallel to the fyzigial line is to the force increafing the gravity of the moon to the earth, as thrice the diftance of the moon from the line of the quadratures to the diftance of the moon from the earth.
Q. E. D.

C O R O L L A R Y.

It is evident from the propofition, that the forces increafing the gravity of the moon to the earth, at different diftances of the moon from the earth, are to one another as the diftances of the moon from the earth; and likewise the forces acting upon the moon in the direction parallel to the fyzigial line, at different diftances of the moon from the line of the quadratures, are to one another

as

as the distances of the moon from the line of the quadratures.

P R O P. IX. *Fig. 9.*

To determine the mean solar force affecting the gravity of the moon to the earth.

Suppose the original orbit of the moon round the earth to be a circle, if the action of the sun did not affect the gravity of the moon to the earth: but it was shown, [Tract. 4. Prop. 10.] that the action of the sun upon the moon, in some parts of the lunar orbit, diminishes the gravity of the moon to the earth, and in other parts of the lunar orbit, increases the gravity of the moon to the earth; but the sum of the forces diminishing the gravity of the moon

to the earth was shown [Cor. to Prop. 4.] to be nearly triple the sum of the forces increasing the gravity of the moon to the earth; and therefore, upon the whole, the action of the sun upon the moon diminishes the gravity of the moon to the earth; and therefore the lunar orbit is lengthened, and the lunar apogee has a progressive motion.

Let TA represent the distance of the moon from the earth, supposing the orbit of the moon to be a circle; let TA likewise represent the centripetal force of the moon to the earth, supposing the action of the sun not to affect the moon; in TA take Am , so that TA may be to Am as the centripetal force of the moon to the earth to the mean solar force affecting the gravity of

of the moon to the earth; and in TA take AG, Aa so that AG is quintuple of Am, and Aa triple of Am; bisect Ga in b; produce TA to P, so that TA may be to TP as TG to Ta; and bisect AP in x. It is evident, from what is demonstrated in the 27th proposition of the fourth tract, that TP will be the greatest distance of the moon from the earth. It is likewise shown, in the same proposition, that the cube of TP will be to the cube of TA in the duplicate ratio of the angle described by the moon from apogee to apogee to four right angles; but the angle described by the moon from apogee to apogee, according to the observations of astronomers, is to four right angles as $363^{\circ} 4' 7'' 30'''$ to 360° ; that is, reducing all to thirds, as 78422850 to 7776000; that is, dividing by 1350, as

58091

58091 to 57600; therefore the cube of TP is to the cube of TA in the duplicate ratio of 58091 to 57600. Suppose TP to TA as the number x to 57600; therefore the cube of the number x will be to the cube of 57600 as the square of 58091 to the square of 57600; therefore thrice the logarithm of the number x , together with twice the logarithm of 57600, will be equal to thrice the logarithm of 57600, together with twice the logarithm of 58091; therefore thrice the logarithm of the number x will be equal to twice the logarithm of 58091, together with the logarithm of 57600; therefore the logarithm of the number x will be equal to two thirds of the logarithm of 58091, together with one third of the logarithm of 57600; but the logarithm of 58091 is 4.76410,88526; therefore

two

two thirds of the logarithm of 58091 is 3.17607,25684; and the logarithm of 57600 is 4.76042,24834; therefore one third of the logarithm of 57600 is 1.58680,74945; therefore two thirds of the logarithm of 58091, together with one third of the logarithm of 57600, is 4.76288,00629; therefore the logarithm of the number x is 4.76288,00629; therefore the number x is 57926.8700349; therefore TP is to TA as 57926.8700349 to 57600; therefore, by division, PA is to AT as 326.8700349 to 57600; therefore, inversely, TA is to AP as 57600 to 326.8700349: but AP is to Ax as 2 to 1; therefore TA is to Ax as 115200 to 326.8700349; therefore, by composition, Tx is to xA as 115526.8700349 to 326.8700349. Because TA is to TP as

TG

TG to Ta , TA is to AP as TG to Ga ;
 therefore TA is to Ax as TG to Gb ;
 therefore Tx is to xA as Tb to bG or ba ;
 therefore Tb is to ba as 115526.8700349 to
 326.8700349; but ba is to bA as 1 to 4;
 therefore Tb is to bA as 115526.8700349
 to 1307.4801396; therefore, by compo-
 sition, TA is to Ab as 116834.3501745 to
 1307.4801396; but Ab is to Am as 4 to 1;
 therefore TA is to Am as 116834.3501745
 to 326.8700349; that is, as 357.43365 to 1;
 but TA is to Am as the centripetal force of
 the moon to the earth to the mean solar
 force affecting the gravity of the moon
 to the earth; therefore the centripetal force
 of the moon to the earth is to the mean
 solar force affecting the gravity of the moon
 to the earth as 357.43365 to 1. *Q. E. D.*

S E C T.

S E C T. II.

The Distance of the Sun from the
Earth determined.

P R O P. X. *Fig. 10.*

LET the circle whose diameter is AB,
and centre T, represent the orbit of
the moon round the earth at T
in TA produced take TV so that TA may
be to TV in the duplicate ratio of the perio-
dic time of the moon round the earth to the
periodic time of the earth round the sun;
in TA take TG so that TV may be to TG
as the centripetal force of the moon to the
earth to the mean solar force affecting the
gravity of the moon to the earth; in TV
G take

take TE so that TG, TA, TE^{*} may be proportionals; and in AV take AD equal to AT; in EB take Eq so that the rectangle contained by AB, Eq may be equal to the rectangle contained by BD, TE; in Bq take Bs so that Bq may be to Bs as Eq, together with AB, to AB; take likewise in Bq, Br so that Bq may be to Br as EB, together with AB, to AB; draw sv, rt perpendicular to AB, meeting the circle in v, t: the ratio of the mean distance of the sun from the earth to the mean distance of the moon from the earth will be greater than the ratio of DB to vs, but less than the ratio of DB to tr.

In TV take TK equal to one third part of the mean distance of the sun from the earth; draw KC a tangent to the circle at C;

C; join TC; draw Tx parallel to KC, meeting the circle in x ; and draw xo perpendicular to AB, meeting AB in o ; in TB take TL so that the rectangle LKC may be equal to the excess of the square of KT above one third part [of the square of TA.

Because the rectangle ATV is to thrice the rectangle KTL [Tract 4. Prop. 14.] as the centripetal force of the moon to the earth to the mean solar force affecting the gravity of the moon to the earth, that is, as TV to TG, that is, as the rectangle ATV to the rectangle ATG; therefore the rectangle ATG is equal to thrice the rectangle KTL; therefore the rectangle EoA is [Tract 4. Prop. 16.] triple of the rectangle ATE: and because the rectangle contained

by AB, Eg , is equal to the rectangle contained by BD, TE, the ratio of Bq to $B\theta$ is greater than the ratio of Eg , together with AB, to AB, but less than the ratio of EB, together with AB, to AB, [Cor 1. and 2. to Prop. 20. Tract 4.]: but Bq is to Br as Eg , together with AB, to AB; and Bq is to Br as EB, together with AB, to AB; therefore the ratio of Bq to $B\theta$ is greater than the ratio of Bq to Br , but less than the ratio of Bq to Br ; therefore $B\theta$ is less than Br , but greater than Br ; therefore $x\theta$ is less than vs , and greater than tr . Again: Because Tx is parallel to KC, the angles θTx , TKC will be equal, and the angles $T\theta x$, KCT are equal, because both right; therefore the triangles $Tx\theta$, KTC are similar; therefore Tx is to $x\theta$ as KT to TC , or TA : but KT is to TA as one third part of the

the mean distance of the sun from the earth to the mean distance of the moon from the earth; therefore Tx is to xo as one third part of the mean distance of the sun from the earth to the mean distance of the moon from the earth; therefore thrice Tx is to xo as the mean distance of the sun from the earth to the mean distance of the moon from the earth: but BD is triple of Tx , or TA ; therefore BD is to xo as the mean distance of the sun from the earth to the mean distance of the moon from the earth: but the ratio of BD to xo is greater than the ratio of BD to vs , and less than the ratio of BD to tr ; therefore the ratio of the mean distance of the sun from the earth to the mean distance of the moon from the earth is greater than the ratio of BD to

vs ,

vs, but less than the ratio of BD to *tr*.
Q. E. D.

P R O P. XI. *Fig. II.*

To determine the ratio of the mean distance of the sun from the earth to the mean distance of the moon from the earth nearly.

Let the circle whose diameter is AB, and centre T, represent the orbit of the moon round the earth at T; in TA produced take TV so that TA may be to TV in the duplicate ratio of the periodic time of the moon round the earth to the periodic time of the earth round the sun; in TA take TG so that TV may be to TG as the centripetal force of the moon to the earth to the mean solar force affecting the gravity

ty of the moon to the earth; in TV take TE so that TG, TA, TE may be proportionals; and in AV take AD equal to AT; in EB take E q so that the rectangle contained by AB, E q may be equal to the rectangle contained by BD, TE; in B q take Bs so that B q may be to Bs as E q , together with AB, to AB; take likewise, in B q , Br so that B q may be to Br as EB, together with AB, to AB; draw sv , rt perpendicular to AB, meeting the circle in v , t .

Because TA is to TV in the duplicate ratio of the periodic time of the moon round the earth to the periodic time of the earth round the sun, that is, as 1 to 178.725, and TV is to TG as the centripetal force of the moon to the earth to the mean solar force affecting the gravity of the moon to the

the earth, that is, [9.], as 357.43365 to 1; therefore, by equality, TA is to TG as 357.43365 to 178.725; but TE is to TA as TA to TG; therefore TE is to TA as 357.43365 to 178.725; therefore TE is to AB as 357.43365 to 357.45; therefore, inversely, AB is to TE as 357.45 to 357.43365. Again: Because the rectangle contained by AB, *Eq* is equal to the rectangle contained by BD, TE, TE will be to *Eq* as AB to BD; that is, as 2 to 3. Because AB is to TE as 357.45 to 357.43365, and TE is to *Eq* as 2 to 3; therefore AB is to *Eq* as 714.9 to 1072.30095; therefore, inversely, *Eq* is to AB as 1072.30095 to 714.9: but AB is to TE as 357.45 to 357.43365; that is, as 714.9 to 714.8673; therefore, by equality, *Eq* is to TE as 1072.30095 to 714.8673;

714.8673; therefore, by division, qT is to TE as 357.43365 to 714.8673; that is, as 1 to 2: but TE is to TA as 357.43365 to 178.725; therefore qT is to TA as 357.43365 to 357.45; therefore, inversely, TA is to Tq as 357.45 to 357.43365; therefore, conversely, TA , or TB , is to Bq as 357.45 to 0.01635; therefore AB is to Bq as 714.9 to 0.01635. Again: Because Eq is to AB as 1072.30095 to 714.9, Eq , together with AB , will be to AB as 1787.20095 to 714.9: but Eq , together with AB , is to AB as Bq to Bs ; therefore Bq is to Bs as 1787.20095 to 714.9. Because AB is to Bq as 714.9 to 0.01635, and Bq is to Bs as 1787.20095 to 714.9; therefore, by perturbate equality, AB is to Bs as 1787.20095 to 0.01635; that is, as 178720095 to 1635; therefore,

H conversely,

conversely, AB is to As as 178720095 to 178718460; therefore the square of AB will be to the rectangle AsB as the square of 178720095 to the product of 178718460 by 1635; that is, as the square of 178720095 to 292204682100: but the square of vs is equal to the rectangle AsB ; therefore the square of AB is to the square of vs as the square of 178720095 to 292204682100; therefore AB is to vs in the subduplicate ratio of the square of 178720095 to 292204682100; that is, as 178720095 to 540559.6; that is, as 330.620518 to 1: but BD is to AB as 3 to 2; therefore BD is to vs as 495.930777 to 1: but the ratio of the mean distance of the sun from the earth to the mean distance of the moon from the earth is [10.] greater than the ratio of DB to vs ; therefore the ratio of the mean distance of the

the

the sun from the earth to the mean distance of the moon from the earth is greater than the ratio of 495.930777 to 1.

Again: Because Eq is to AB as 1072.30095 to 714.9, and AB is to Bq as 714.9 to 0.01635; therefore, by equality, Eq is to Bq as 1072.30095 to 0.01635; therefore, by composition, EB is to Bq as 1072.3173 to 0.01635: but Bq is to AB as 0.01635 to 714.9; therefore, by equality, EB is to AB as 1072.3173 to 714.9; therefore, by composition, EB , together with AB , is to AB as 1787.2173 to 714.9: but EB , together with AB , is to AB as Bq to Br ; therefore Bq is to Br as 1787.2173 to 714.9. Because AB is to Bq as 714.9 to 0.01635, and Bq to Br as 1787.2173 to 714.9; therefore, by perturbate e-

H 2

quality,

quality, AB is to Br as 1787.2173 to 0.01635 , that is, as 178721730 to 1635 ; therefore, conversely, AB is to Ar as 178721730 to 178720095 ; therefore the square of AB will be to the rectangle ArB as the square of 178721730 to the product of 178720095 by 1635 ; that is, as the square of 178721730 to 292207355325 : but the square of tr is equal to the rectangle ArB ; therefore the square of AB is to the square of tr as the square of 178721730 to 292207355325 ; therefore AB is to tr in the subduplicate ratio of the square of 178721730 to 292207355325 , that is, as 178721730 to 540562.073 , that is, as 330.622 to 1 : but BD is to AB as 3 to 2 ; therefore BD is to tr as 495.933 to 1 : but the ratio of the mean distance of the sun from the

earth

earth to the mean distance of the moon from the earth is [10.] less than the ratio of BD to tr ; therefore the ratio of the mean distance of the sun from the earth to the mean distance of the moon from the earth is less than the ratio of 495.933 to 1. It was shown in the preceding paragraph, that the ratio of the mean distance of the sun from the earth to the mean distance of the moon from the earth is greater than the ratio of 495.930777 to 1; and it is shown just now, that the ratio of the mean distance of the sun from the earth to the mean distance of the moon from the earth is less than the ratio of 495.933 to 1; therefore, taking the mean of these two, the ratio of the mean distance of the sun from the earth to the mean distance of the moon from the earth will be nearly equal to the

the ratio of 495.9315 to 1: for, taking half the difference of the extremes, the error is less than 0.0011115; therefore the error is less than the 430000 part of the real distance. Q. E. I.

P R O P. XII.

To determine the proportion of the diameter of the sun to the diameter of the earth; and likewise the proportion of the magnitude of the sun to the magnitude of the earth and moon.

The mean apparent diameter of the sun, according to Sir Isaac Newton, is $32' 12''$, and the mean apparent diameter of the moon is $31' 16'' 30'''$; therefore the mean apparent diameter of the sun is to the mean
apparent

apparent diameter of the moon as 1288 to 1251: but the apparent diameter of the moon at the distance of the moon from the earth, is to the apparent diameter of the moon at the distance of the sun from the earth, as the distance of the sun from the earth to the distance of the moon from the earth, that is, as 495.9315 to 1; therefore the apparent diameter of the sun will be to the apparent diameter of the moon at the distance of the sun from the earth, as 638759.772 to 1251, that is, [dividing by 9], as 70973.308 to 139; therefore the diameter of the sun is to the diameter of the moon as 70973.308 to 139: but the diameter of the moon [according to Sir Isaac Newton] is to the mean diameter of the earth as 100 to 365, that is, as 20 to 73; therefore the diameter of the sun is to the diameter

diameter of the earth as 1419466.16 to 10147, that is, as 139.89 to 1: and the magnitude of the sun is to the magnitude of the earth as the cube of the diameter of the sun to the cube of the diameter of the earth; therefore the magnitude of the sun will be to the magnitude of the earth as 2737537.08 to 1.

Again: Because the diameter of the sun is to the diameter of the moon as 70973.308 to 139, that is, as 510.599 to 1, and the magnitude of the sun is to the magnitude of the moon as the cube of the diameter of the sun to the cube of the diameter of the moon; therefore the magnitude of the sun will be to the magnitude of the moon as 133118948.88 to 1. *Q. E. I.*

P R O P.

P R O P. XIII.

To determine the mean distance of the sun from the earth in semidiameters of the earth ; likewise to determine the parallax of the sun, that is, to determine the angle which the semidiameter of the earth would subtend if viewed from the sun.

Because the apparent diameter of the sun is [according to Sir Isaac Newton] $32' 12''$, the apparent semidiameter of the sun will be $16' 6''$; therefore the semidiameter of the sun will be to the distance of the sun from the earth as the sine of $16' 6''$ to the radius, that is, as 1 to 213.56; therefore, inversely, the distance of the sun from the earth will be to the semidiameter of the sun

I

as

as 213.56 to 1 : but the semidiameter of the sun is to the semidiameter of the earth as 139.89 to 1 ; therefore the distance of the sun from the earth will be to the semidiameter of the earth as 29874.9 to 1 ; therefore the mean distance of the sun from the earth will contain 29874.9 mean semidiameters of the earth.

Again : Because the apparent semidiameter of the sun is 16' 6", that is, 966", and the semidiameter of the sun is to the semidiameter of the earth as 139.89 to 1 ; therefore as 139.89 is to 1, so is 966" to the number of seconds in the angle subtended by the semidiameter of the earth if viewed from the sun ; and therefore the angle which the semidiameter of the earth would
subtend

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subtend if viewed from the sun, will be $6''.9$, that is, $6''\ 54'''$. Q. E. I.

C O R O L L A R Y I.

It was shown in Prop. 11. that the distance of the sun from the earth was to the distance of the moon from the earth, nearly as 495.9315 to 1; therefore the mean distance of the moon from the earth will be 60.24 mean semidiameters of the earth; which agrees very nearly with the distance assigned by Sir Isaac Newton.

C O R O L L A R Y II.

Because the semidiameter of the sun is to the distance of the sun from the earth as 1 to 213.56, and the distance of the sun from

I 2

the

the earth is to the distance of the moon from the earth as 495.9315 to 1; therefore the semidiameter of the sun is to the distance of the moon from the earth as 495.9315 to 213.56, that is, nearly, as 55.1039 to 23.73: but the magnitude of the sun will be to the magnitude of a globe whose semidiameter is equal to the distance of the moon from the earth as the cube of 55.1039 to the cube of 23.73; therefore the magnitude of the sun will be to the magnitude of a globe whose semidiameter is equal to the distance of the moon from the earth as 12.52 to 1.

S C H O L I U M.

The ingenious Mr Machin, late professor of astronomy in Gresham college, in his excellent

cellent treatise, intituled, *The laws of the moon's motion according to gravity*, gives the following rule, in the 22d page, for determining the mean motion of the sun from the lunar node.

“ The mean motion of the sun from the
“ node is a geometrical mean proportional
“ between the motion of the sun and the
“ mean motion of the sun from the node,
“ in the month when the line of the nodes
“ is in quadrature with the sun.

“ How near this rule agrees with the ob-
“ servations, will appear by this calculation.

“ Since the mean motion of the node,
“ in that month when the line of the nodes
“ is in quadrature to the sun, was before
“ shown to be to the moon's mean motion

“ as

“ as 10 to 1211; and the motion of the sun
“ is to the motion of the moon as 160 to
“ 2139; it follows, that the motion of the
“ node, and the motion of the sun, will be
“ in the proportion of 154 and 1395; and
“ therefore, by the rule, the sun's mean mo-
“ tion from the node is to the sun's mean
“ motion in the subduplicate proportion of
“ 1549 to 1395, that is, nearly as 98 to
“ 93; which corresponds with the observa-
“ tions, there being 98 revolutions of the
“ sun to the node in 93 revolutions of the
“ sun. The subduplicate proportion taken
“ more nearly is as 941 to 893, which will
“ produce $19^{\circ} 21' 3''$ for the motion of
“ the node from the fixed stars in a syde-
“ real year. The motion [as observed] is
“ $19^{\circ} 21' 22''$.

“ Had

“ Had the calculation from the rule been
 “ more exactly made in large numbers,
 “ the annual motion produced would be
 “ $19^{\circ} 21' 7\frac{1}{2}''$, which is $14''$ less than the
 “ motion as observed by the astronomers.

“ Which difference may very probably
 “ arise from the sun's parallax; and if so,
 “ it may perhaps furnish the best and most
 “ certain method of adjusting and fixing
 “ the true distance of the sun. For the
 “ sun's force being something more on
 “ that half of the orb which is towards
 “ the sun, than what it is on the other half,
 “ the elliptic epicycle is accordingly larger
 “ in the first case than in the latter: and,
 “ by calculation, I find, that the mean
 “ motion of the node arising, after con-
 “ sideration is had of this difference, is more
 “ than

“ than the mean motion from the mean
“ magnitude of the epicycle by near 2" in
“ the year, for every minute in the paral-
“ lactic angle of the orbit of the moon,
“ or for every second of the sun's parallax :
“ and, by the best computation I have yet
“ made, this difference of 14" in the an-
“ nual motion of the node, will arise from
“ about 8" of parallax, which will make
“ the sun's distance above 25000 semi-
“ diameters of the earth.”

It is evident from the above quotation, that this eminent mathematician suspected that it was possible to determine the sun's distance from the earth by the theory of gravity. By his method he found, that the mean motion of the node is more than the mean motion from the mean magnitude of the

the

the epicycle by near 2" in the year for every second of the sun's parallax: by this method, therefore, the difference of 14" in the annual motion of the node will arise from about 7" of parallax; which agrees very nearly to what we have above demonstrated it to be. The celebrated Mr Huygens, in his ingenious treatise, intituled, *Cosmotheoros; or, Conjectures concerning the planetary worlds*, asserts, that the distance of the sun from the earth is not less than 24000 semidiameters of the earth; and the very ingenious and accurate Monf. de la Hire in his *Tabul. Astron.* [p. 6. of the *Tables*] thinks the sun's horizontal parallax to be not above 6"; and his distance therefore to be 34377 semidiameters of the earth. Both these astronomers make the distance of the sun from the earth to be greater than what was generally

K imagined,

imagined. The distance we have assigned is nearly a mean between both.

P R O P. XIV.

To determine the mean distance of the sun and moon from the earth in English miles : likewise to determine the semidiameter of the sun and moon in English miles.

An arc of one degree of a meridian of the earth [upon the supposition that the earth is of a spherical figure] according to Mr Picart's mensuration, contains 57060 Paris toises; that is, 365658 English feet; that is, 69.2534 English miles; therefore a great circle of the earth will contain 24931.224 English miles; therefore the
semidiameter

Prop. 14. FROM THE EARTH. 75

femidiameter of the earth will contain
3967.9272 English miles.

Because the distance of the sun from the
earth is [13.] 29874.9 mean femidiameters
of the earth ; therefore the distance of the
sun from the earth will be 118541428
English miles.

Because the mean distance of the sun
from the earth is [11.] to the mean distance
of the moon from the earth as 495.9315 to
1 ; therefore the mean distance of the moon
from the earth will be 239027.9 English
miles.

Again : Because the femidiameter of the
sun is to the femidiameter of the earth as
139.89 to 1 ; therefore the femidiameter of

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the

the sun will contain 555073 English miles.

Because the femidiameter of the earth is to the femidiameter of the moon as 365 to 100, that is, as 73 to 20, and the femidiameter of the earth contains 3967.9272 English miles; therefore the femidiameter of the moon will contain 1087 English miles. Q. E. I.

P R O P. XV.

To determine the proportion of the quantity of matter in the sun to the quantity of matter in the earth and moon; as also the proportion of the density of the sun to the density of the earth: likewise to determine the ratio of the forces of the sun and moon upon the tides.

The duplicate ratio of the periodic time
of

of the moon round the earth, were the moon at the same distance from the earth that the sun is, to the periodic time of the moon round the earth, is as the cube of the distance of the sun from the earth to the cube of the distance of the moon from the earth, that is, as the cube of 495.9315 to 1, that is, as 121973386.6937 to 1; and the duplicate ratio of the periodic time of the moon round the earth to the periodic time of the earth round the sun is as 1 to 178.725; therefore the duplicate ratio of the periodic time of the moon round the earth, were the moon at the same distance from the earth that the sun is, to the periodic time of the earth round the sun, is as 121973386.6937 to 178.725, that is, as 682464 to 1; and therefore [Cor. to Prop. 5. Tract 4.] the attractive force of the sun is to the

the attractive force of the earth as 682464 to 1; therefore the quantity of matter in the sun is to the quantity of matter in the earth as 682464 to 1.

The quantity of matter in a globe of the same magnitude with the sun, and of the same density with the earth, will be to the quantity of matter in the earth as the magnitude of the sun to the magnitude of the earth, that is, as 2737537.08 to 1; and the quantity of matter in the earth is to the quantity of matter in the sun as 1 to 682464; therefore the quantity of matter in a globe of the same magnitude with the sun, and of the same density with the earth, will be to the quantity of matter in the sun as 2737537.08 to 682464; therefore the density of the earth is to the density of the sun

sun as 2737537.08 to 682464, that is, as 4 to 1 nearly.

It is generally allowed by astronomers, that the nearer the planets are to the sun their densities are greater; and as the moon may be considered to be at the same distance from the sun that the earth is, the density of the moon may be considered to be equal to the density of the earth; and because the diameter of the moon is to the diameter of the earth as 100 to 365, that is, as 20 to 73; therefore the mass of matter in the moon will be to the mass of matter in the earth as 8000 to 389017, that is, as 1 to 48.627: but the mass of matter in the earth is to the mass of matter in the sun as 1 to 682464; therefore the mass of matter in the moon will be to the mass of matter in

in the sun as 1 to 33186176.928; therefore the force of the moon upon the tides will be to the force of the sun upon the tides, were the sun at the same distance from the earth that the moon is, as 1 to 33186176.928: but the force of the sun to move the tides, were the sun at the same distance from the earth that the moon is, is to the force of the sun to move the tides in the triplicate ratio of the distance of the sun from the earth to the distance of the moon from the earth, that is, as 121973386.6937 to 1; therefore the force of the moon to move the tides will be to the force of the sun to move the tides as 121973386.6937 to 33186176.928, that is, nearly as 3.6754 to 1.

Sir Isaac Newton, in the 37th proposition
of

of the third book of his *Principia*, makes the force of the moon to move the sea to be to the force of the sun to move the sea as 4.4815 to 1. Mr Daniel Bernouillie, in his treatise on the flux and reflux of the sea, which gained the premium from the academy 1740, makes the force of the moon to move the sea to be to the force of the sun to move the sea as 2.5 to 1. Sir Isaac Newton and Mr Bernouillie deduce their proportions from some observations of the height of the tides. Mr Bernouillie alledges, that the observations Sir Isaac Newton made use of to fix his proportion are ill chosen. We will not pretend to give a positive decision which of their observations were best chosen; but as the height of the tides is affected by a variety of circumstances, such as the depth, the situation, and the

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different

different windings of the coasts, the currents, winds, &c. which render it impossible to determine, with any degree of accuracy, the forces of the sun and moon to affect the tides; therefore these observations are not to be depended upon. The celebrated M. d'Alembert, in his ingenious treatise on the precession of the equinoxes, and the nutation of the earth's axis, published at Paris in the year 1749, by a different, but very accurate method, makes the force of the moon to move the sea to be to the force of the sun to move the sea nearly as 19 to 6, that is, nearly as 3.1666 to 1; which proportion agrees nearly with the proportion before assigned, supposing the moon's density to be the same with the density of the earth.

Some

Some imagine that there are no fluids in the moon, because there is no atmosphere observed about the moon. If this was the case, it is reasonable to suppose the density of the moon to be greater than the density of the earth. But supposing the moon to be a globe of the same texture with the globe of the earth, and to be furnished with fluids in proportion to those of the earth, the height of the atmosphere of the moon would be so small, as that it could not be observed by the most nice observations. It will be allowed, that the height of the atmosphere will be in proportion to the velocity of the moon round its axis, and the quantity of fluids on its surface. The velocity of the moon round its axis is less than the twenty-seventh part of the velocity of the earth round its axis; and the quantity of fluids

on the surface of the moon will be less than the twelfth part of the quantity of fluids on the surface of the earth; therefore the height of the atmosphere of the moon would be very little in comparison of the height of the atmosphere of the earth. Supposing the height of the atmosphere of the earth to be about 50 miles, the height of the moon's atmosphere would be less than the sixth part of a mile; which, if viewed from the earth, would subtend an angle less than the sixth part of a second. The reason assigned by some astronomers for alledging, that the moon has no atmosphere, is, That if the moon had an atmosphere, the planets and stars which often are seen near its limb (and sometimes the moon passes over them) would have their light refracted. But in answer to this, it is to be observed, that during the transit

transit of the moon over a planet or fixed star, the time of the transit of the atmosphere of the moon would be less than the third part of a second of time; which time is so small, that no astronomer can pretend to observe it.

P R O P. XVI.

To determine the distances of the planets from the sun in semidiameters of the earth, and likewise in English miles.

The periodic times of the planets (with respect to the fixed stars), revolving about the sun, are, in days, and decimal parts of a day, according to Sir Isaac Newton, as follows.

Saturn;

Saturn,	10759.275
Jupiter,	4332.514
Mars,	686.9785
Earth,	365.2565
Venus,	224.6176
Mercury,	87.9692

and therefore, because the squares of the periodic times are as the cubes of their mean distances from the sun, the mean distances of Saturn, Jupiter, Mars, Venus, Mercury, from the sun, will be to the mean distance of the earth from the sun as the following numbers.

Saturn,	9540061	to 100000
Jupiter,	520096	
Mars,	152369	
Venus,	72333	
Mercury,	38710	

Because

Prop. 16. FROM THE EARTH. 87

Because the mean distance of the earth from the sun contains 29874.9 mean semidiameters of the earth; therefore the distances of Saturn, Jupiter, Mars, Venus, Mercury, from the sun, in mean semidiameters of the earth, will be as follows.

Saturn,	285008
Jupiter,	155378
Mars,	45520
Venus,	21609
Mercury,	11563

Because the semidiameter of the earth contains 3968 English miles nearly; therefore the distances of Saturn, Jupiter, Mars, Venus, Mercury, from the sun, in English miles, will be as follows.

Saturn,

Saturn,	1130911644
Jupiter,	616539904
Mars,	180623360
Venus,	85744512
Mercury,	45881984

The distances of the planets from the sun being determined in English miles, their distances in Scots miles will be easily computed; thirty-three Scots miles being equal to thirty-seven English miles.

P R O P. XVII.

To determine the velocity of the moon and earth in their respective orbits.

Because the semidiameter of the earth is
[Cor. 1. Prop. 13.] to the mean distance of
the

the moon from the earth as 1 to 60.24; therefore a degree of a great circle of the earth will be to a degree of the lunar orbit as 1 to 60.24: but a degree of a great circle of the earth contains 69.2534 English miles; therefore a degree of the lunar orbit will contain 4171.8248 English miles. Because the periodic time of the moon round the earth is 27 days 7 hours 43 minutes, that is, 39343'; therefore 39343' is to 1' as 360° to the arc of the lunar orbit described by the moon in a minute: but 360° contains 21600'; therefore 39343 is to 1 as 21600' to 0.549'; therefore the arc of the lunar orbit described by the moon in a minute is 0.549'. Because 60', that is, a degree of the lunar orbit, contains 4171.8248 English miles; therefore as 60 is to 0.549 so is 4171.8248 to 38.1721969, the num-

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ber of English miles the moon passes through in a minute of time.

Again : Because the femidiameter of the earth is [13.] to the mean distance of the sun from the earth as 1 to 29874.9 ; therefore a degree of a great circle of the earth will be to a degree of the earth's orbit round the sun as 1 to 29874.9 : but a degree of a great circle of the earth contains 69.2534 English miles ; therefore a degree of the earth's orbit will contain 2067938.39966 English miles. Because the periodic time of the earth round the sun is 365 days 6 hours 9 minutes, that is, 526029' ; therefore 526029 is to 1 as 360° , that is, 21600', to 0.04106239' ; therefore the arc of the annual orbit described by the earth in a minute of time is 0.04106239'. Because 60',
that

Prop. 18. FROM THE EARTH. 91

that is, a degree of the annual orbit, contains 2067938.39966 English miles; therefore as 60' is to 0.04106239', so is 2067938.39966 to 1415.24, the number of English miles the earth passes through in a minute of time.

The number of miles the earth passes through in a minute of time being determined, the number of miles any of the planets passes through in a minute of time will be easily determined by the following proposition.

P R O P. XVIII. Fig. 12.

The velocities of any two planets round the sun are reciprocally in the subduplicate ratio of their distances from the sun.

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Let

Let the circles ABC , DEF represent the orbits of two planets round the sun at S , the centre of both circles; let AG , DH be parts of the orbits described in an indefinitely little part of time; join SA , SD , meeting the circles ABC , DEF in C , F ; join AG , DH , and CG , FH ; and draw GK , HL perpendicular to AC , DF , meeting AC , DF in K , L .

Because the arcs AG , DH are described in an indefinitely little part of time, the planet revolving in the circle ABC would, by the centripetal force tending to the sun, descend through a space equal to AK in the time the arc AG is described; likewise the planet revolving in the circle DEF would descend through a space equal to DL in the time the arc DH is described: but the arcs
 AG ,

AG, DH are described in equal times; therefore the spaces AK, DL would be passed over in equal times; therefore AK will be to DL as the centripetal force at A to the centripetal force at D, that is, as the square of SD to the square of SA, that is, as the square of FD to the square of CA: but AK is to DL as the rectangle CAK to the rectangle AC,DL; therefore the rectangle CAK is to the rectangle AC,DL as the square of FD to the square of AC; therefore, alternating, the rectangle CAK is to the square of FD as the rectangle AC,DL to the square of AC, that is, as DL to AC: but DL is to AC as the rectangle FDL to the rectangle FD,AC; therefore the rectangle CAK is to the square of FD as the rectangle FDL to the rectangle FD,AC; therefore, alternating, the rectangle CAK
is

is to the rectangle FDL as the square of FD to the rectangle FD, AC, that is, as FD to AC, that is, as SD to SA. But because AG, DH are indefinitely little, the rectangle CAK may be considered as equal to the square of AG, and the rectangle FDL may be considered as equal to the square of DH; therefore the square of AG will be to the square of DH as SD to SA; therefore AG will be to DH in the subduplicate ratio of SD to SA: but because the arcs AG, DH are described in the same time, the velocity of the planet describing the arc AG will be to the velocity of the planet describing the arc DH, as the arc AG to the arc DH; therefore the velocity of the planet revolving in the circle ABC will be to the velocity of the planet revolving in the circle DEF in the subduplicate ratio of SD to SA. *Q. E. D.*

P R O P.

P R O P. XIX.

To determine the velocity of Saturn, Jupiter, Mars, Venus, and Mercury, in their respective orbits.

The distance of Saturn from the sun is to the distance of the earth from the sun as 954006 to 100000; therefore the velocity of the earth in its orbit will be to the velocity of Saturn in his orbit in the subduplicate ratio of 954006 to 100000, that is, nearly as 3088 to 1000; therefore as 3088 to 1000 so is 1415.24, the number of English miles the earth passes through in a minute, to 458, the number of English miles that Saturn passes through in a minute of time.

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The distance of Jupiter from the sun is to the distance of the earth from the sun as 520096 to 100000; therefore the velocity of the earth in its orbit will be to the velocity of Jupiter in his orbit in the subduplicate ratio of 520096 to 100000, that is, nearly as 228 to 100; therefore as 228 to 100, so is 1415.24, the number of English miles the earth passes through in a minute, to 620, the number of English miles that Jupiter passes through in a minute of time.

The distance of Mars from the sun is to the distance of the earth from the sun as 152369 to 100000; therefore the velocity of the earth in its orbit will be to the velocity of Mars in his orbit in the subduplicate ratio of 152369 to 100000, that is, nearly as 1234 to 1000; therefore as 1234 to
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Fig. 6.

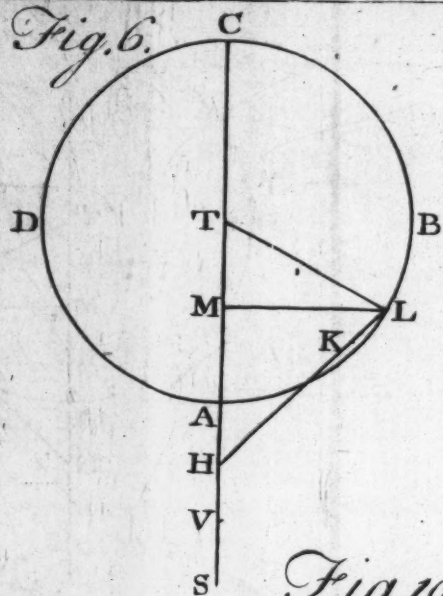


Fig. 7.

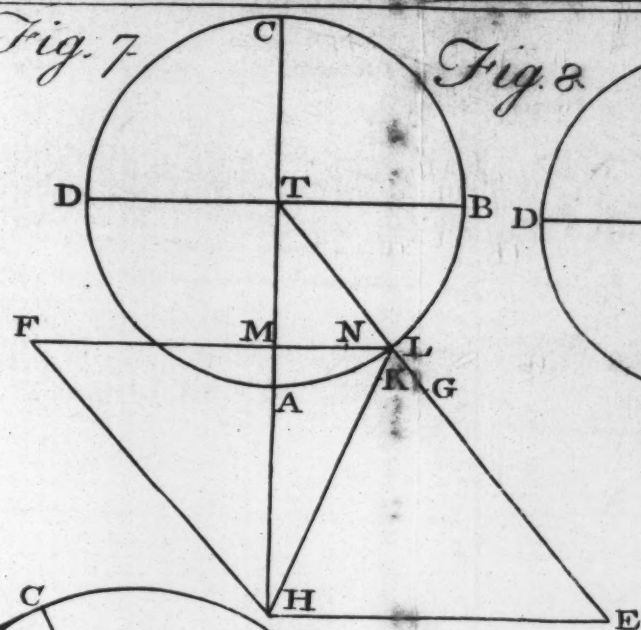


Fig. 8.

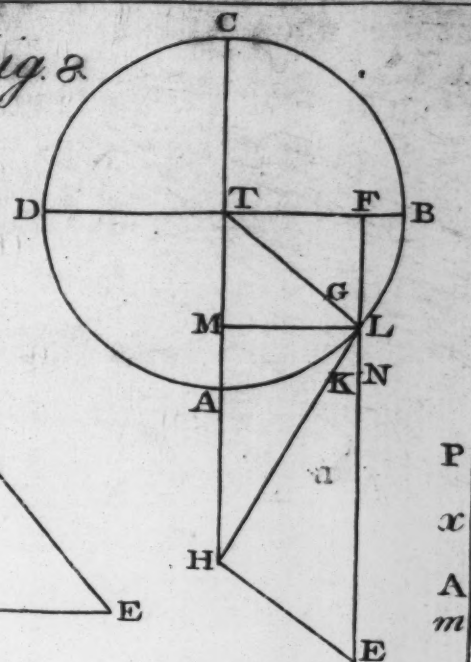


Fig. 10.

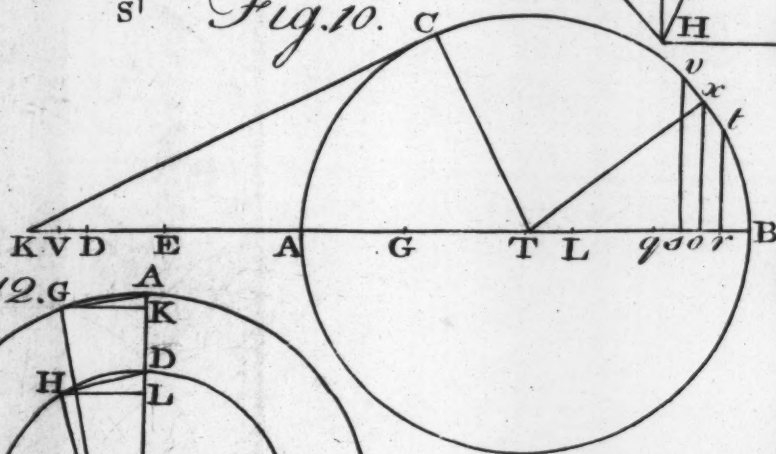


Fig. 12.

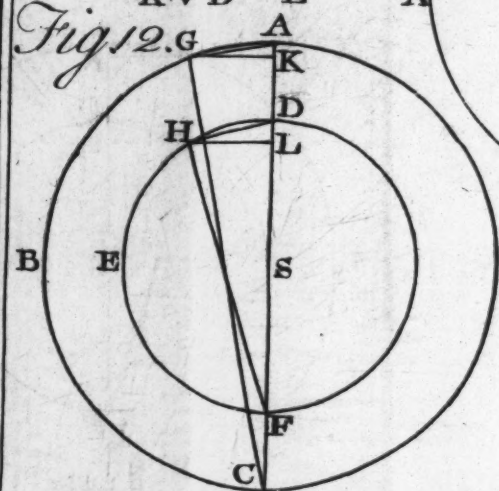


Fig. 11.

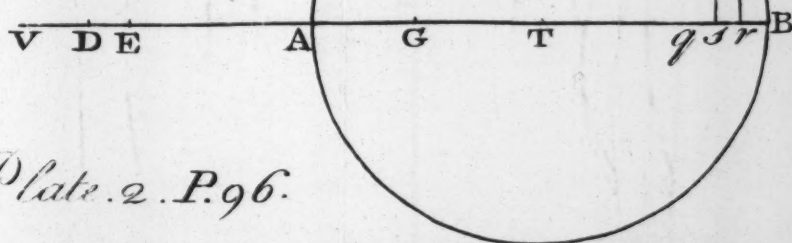


Fig. 9.

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1000, so is 1415.24, the number of English miles the earth passes through in a minute, to 1146, the number of English miles that Mars passes through in a minute of time.

The distance of Venus from the sun is to the distance of the earth from the sun as 72333 to 100000, therefore the velocity of the earth in its orbit will be to the velocity of Venus in her orbit in the subduplicate ratio of 72333 to 100000, that is, nearly as 17 to 20; therefore as 17 is to 20, so is 1415.24, the number of English miles the earth passes through in a minute, to 1665, the number of English miles that Venus passes through in a minute of time.

The distance of Mercury from the sun is to the distance of the earth from the sun

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as 38710 to 100000 ; therefore the velocity of the earth in its orbit will be to the velocity of Mercury in his orbit in the subduplicate ratio of 38710 to 100000, that is, nearly as 622 to 1000 ; therefore as 622 to 1000, so is 1415.24, the number of English miles the earth passes through in a minute, to 2275, the number of English miles that Mercury passes through in a minute of time.

P R O P. XX.

To determine the magnitude of Mercury, Venus, Mars, Jupiter, and Saturn.

The diameters of Mercury, Venus, Mars, Jupiter, and Saturn, seen from the sun at the

the mean distances, (according to the celebrated M. de la Caille, in his *Elements of Astronomy*), are as follows.

Mercury,	21"
Venus,	30"
Mars,	12"
Jupiter,	37"
Saturn,	16"

Therefore the diameters of Mercury, Venus, Mars, Jupiter, and Saturn, seen from the sun at the distance of the earth from the sun, will be as follows.

Mercury,	8"	7'''
Venus,	21"	42'''
Mars,	18"	17'''
Jupiter,	3' 12"	26'''
Saturn,	2' 32"	38'''

Therefore the diameters of Mercury, Venus, Mars, Jupiter, and Saturn, will be to the diameter of the earth as the following numbers.

Mercury,	58816	
Venus,	157246	
Mars,	132487	to 100000
Jupiter,	1393719	
Saturn,	1106038	

And because the diameter of the earth contains nearly 7936 English miles; therefore the diameters of Mercury, Venus, Mars, Jupiter, and Saturn, in English miles, will be as follows.

Mercury,	4667
Venus,	12479
Mars,	10514
Jupiter,	110605
Saturn,	87775

The

The magnitude of Mercury, Venus, Mars, Jupiter, and Saturn, will be to the magnitude of the earth as the following numbers.

Mercury,	0.20346	}	to 1
Venus,	3.88811		
Mars,	2.32551		
Jupiter,	2707		
Saturn,	1353		

Therefore the magnitude of Mercury is a little more than the fifth part of the magnitude of the earth.

The magnitude of Venus is nearly four times the magnitude of the earth.

The magnitude of Mars is nearly equal to twice the magnitude of the earth, together
with

with one third part of the magnitude of the earth.

The magnitude of Jupiter is nearly two thousand seven hundred and seven times the magnitude of the earth.

The magnitude of Saturn is nearly one thousand three hundred and fifty-three times the magnitude of the earth.

P R O P. XXI.

To determine the proportion of the quantity of matter in Jupiter and Saturn, to the quantity of matter in the earth.

The density of Jupiter is to the density of
the

Prop. 21. FROM THE EARTH. 103

the earth (according to Sir Isaac Newton) as 94.5 to 400, and the magnitude of Jupiter is to the magnitude of the earth as 2707 to 1; therefore the quantity of matter in Jupiter will be to the quantity of matter in the earth as 639 to 1.

The density of Saturn is to the density of the earth (according to Sir Isaac Newton) as 67 to 400, and the magnitude of Saturn is to the magnitude of the earth as 1353 to 1; therefore the quantity of matter in Saturn will be to the quantity of matter in the earth as 226 to 1.

F I N I S.

the earth (according to Sir Isaac Newton) as 94 to 1000, and the magnitude of Jupiter is to the magnitude of the earth as 2707 to 1; therefore the density of matter in Jupiter will be to the quantity of matter in the earth as 699 to 1.

The density of Saturn is to the density of the earth (according to Sir Isaac Newton) as 67 to 1000, and the magnitude of Saturn is to the magnitude of the earth as 2304 to 1; therefore the quantity of matter in Saturn will be to the quantity of matter in the earth as 226 to 1.

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